C Converting between grid eastings and northings and ellipsoidal latitude and longitude

Converting latitude and longitude to eastings and northings

To convert a position from the graticule of latitude and longitude coordinates (ϕ,λ) to a grid of easting and northing coordinates (E, N) using a Transverse Mercator projection (for example, National Grid or UTM), compute the following formulae. Remember to express all angles in radians. You will need the ellipsoid constants a, b and e^2 and the following projection constants. Annexe A gives values of these constants for the ellipsoids and projections usually used in Britain.

N_0	northing of true origin
E_0	easting of true origin
F_0	scale factor on central meridian
ϕ_0	latitude of true origin
λ_0	longitude of true origin and central meridian.

$$n = \frac{a-b}{a+b} \qquad \text{C1}$$

$$\nu = aF_0(1-e^2\sin^2\phi)^{-0.5}$$

$$\rho = aF_0(1-e^2)(1-e^2\sin^2\phi)^{-1.5}$$

$$\eta^2 = \frac{\nu}{\rho} - 1 \qquad \text{C2}$$

$$M = bF_0\left(\begin{array}{l}\left(1+n+\frac{5}{4}n^2+\frac{5}{4}n^3\right)(\phi-\phi_0) - \left(3n+3n^2+\frac{21}{8}n^3\right)\sin(\phi-\phi_0)\cos(\phi+\phi_0) \\ +\left(\frac{15}{8}n^2+\frac{15}{8}n^3\right)\sin(2(\phi-\phi_0))\cos(2(\phi+\phi_0)) - \frac{35}{24}n^3\sin(3(\phi-\phi_0))\cos(3(\phi+\phi_0))\end{array}\right)$$

C3

$$\text{I} = M + N_0$$

$$\text{II} = \frac{\nu}{2}\sin\phi\cos\phi$$

$$\text{III} = \frac{\nu}{24}\sin\phi\cos^3\phi(5-\tan^2\phi+9\eta^2)$$

$$\text{IIIA} = \frac{\nu}{720}\sin\phi\cos^5\phi(61-58\tan^2\phi+\tan^4\phi)$$

$$\text{IV} = \nu\cos\phi$$

$$\text{V} = \frac{\nu}{6}\cos^3\phi\left(\frac{\nu}{\rho}-\tan^2\phi\right)$$

$$\text{VI} = \frac{\nu}{120}\cos^5\phi\left(5-18\tan^2\phi+\tan^4\phi+14\eta^2-58(\tan^2\phi)\,\eta^2\right)$$

$$N = \text{I} + \text{II}(\lambda-\lambda_0)^2 + \text{III}(\lambda-\lambda_0)^4 + \text{IIIA}(\lambda-\lambda_0)^6 \qquad \text{C4}$$

$$E = E_0 + \text{IV}(\lambda-\lambda_0) + \text{V}(\lambda-\lambda_0)^3 + \text{VI}(\lambda-\lambda_0)^5 \qquad \text{C5}$$

Here's a worked example using the Airy 1830 ellipsoid and National Grid. Intermediate values are shown here to 10 decimal places. Compute all values using double-precision arithmetic.

ϕ	52° 39′ 27.2531″ N	III	1.5606875424E+05
λ	1° 43′ 4.5177″ E	IIIA	-2.0671123011E+04
		IV	3.8751205749E+06
ν	6.3885023333E+06	V	-1.7000078208E+05
ρ	6.3727564399E+06	VI	-1.0134470432E+05
η^2	2.4708136169E-03		
M	4.0668829596E+05	E	651409.903 m
I	3.0668829596E+05	N	313177.270 m
II	1.5404079092E+06		

Converting eastings and northings to latitude and longitude

Obtaining (λ, ϕ) from (E, N) is an iterative procedure. You need values for the ellipsoid and projection constants a, b, e^2, N_0, E_0, F_0, ϕ_0, and λ_0 as in the previous section. Remember to express all angles in radians. First compute

$$\phi' = \left(\frac{N - N_0}{aF_0}\right) + \phi_0 \qquad \text{C6}$$

and M from equation (C3), substituting ϕ' for ϕ. If the absolute value of $(N - N_0 - M) \geq 0.01\ \text{mm}$, obtain a new value for ϕ' using

$$\phi'_{new} = \left(\frac{N - N_0 - M}{aF_0}\right) + \phi' . \qquad \text{C7}$$

and recompute M substituting ϕ' for ϕ. Iterate until the absolute value of $(N - N_0 - M) < 0.01\ \text{mm}$, then compute ρ, ν and η^2 using equation (C2) and compute

$$\text{VII} = \frac{\tan\phi'}{2\rho\nu}$$

$$\text{VIII} = \frac{\tan\phi'}{24\rho\nu^3}\left(5 + 3\tan^2\phi' + \eta^2 - 9(\tan^2\phi')\eta^2\right)$$

$$\text{IX} = \frac{\tan\phi'}{720\rho\nu^5}\left(61 + 90\tan^2\phi' + 45\tan^4\phi'\right)$$

$$\text{X} = \frac{\sec\phi'}{\nu}$$

$$\text{XI} = \frac{\sec\phi'}{6\nu^3}\left(\frac{\nu}{\rho} + 2\tan^2\phi'\right)$$

$$\text{XII} = \frac{\sec\phi'}{120\nu^5}\left(5 + 28\tan^2\phi' + 24\tan^4\phi'\right)$$

$$\text{XIIA} = \frac{\sec\phi'}{5040\nu^7}\left(61 + 662\tan^2\phi' + 1320\tan^4\phi' + 720\tan^6\phi'\right)$$

$$\phi = \phi' - \text{VII}(E - E_0)^2 + \text{VIII}(E - E_0)^4 - \text{IX}(E - E_0)^6 \qquad \text{C8}$$

$$\lambda = \lambda_0 + \text{X}(E - E_0) - \text{XI}(E - E_0)^3 + \text{XII}(E - E_0)^5 - \text{XIIA}(E - E_0)^7 \qquad \text{C9}$$

Here’s a worked example using the Airy 1830 ellipsoid and National Grid. Intermediate values are shown here to 10 decimal places. Compute all values using double-precision arithmetic.

E	651409.903 m	η^2	2.4642205195E-03
N	313177.270 m	VII	1.6130562490E-14
ϕ'	9.2002324604E-01 rad	VIII	3.3395547442E-28
M	4.1290347144E+05	IX	9.4198561719E-42
2nd ϕ'	9.2006619470E-01 rad	X	2.5840062509E-07
M	4.1317717541E+05	XI	4.6985969968E-21
3rd ϕ'	9.2006620953E-01 rad	XII	1.6124316622E-34
M	4.1317726997E+05	XIIA	6.6577316330E-48
final ϕ'	9.2006620954E-01 rad		
ν	6.3885233415E+06	ϕ	52° 39′ 27.2531″ N
ρ	6.3728193094E+06	λ	1° 43′ 4.5177″ E